Key

3.10 Fundamental Identities

Name_____ **Date** _____ **Period** _____

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Use basic identities to simplify the expression. Show work!

1) $\tan\theta \cos\theta$ $\frac{\sin\theta}{\cos\theta} \cdot \frac{\cos\theta}{1} = \sin\theta$ 1) $\sin\theta$

2) $\sec\theta \sin(\frac{\pi}{2} - \theta)$ $\frac{1}{\cos\theta} \cdot \frac{\cos\theta}{1} = 1$ 2) 1

3) $\frac{1+\tan^2\theta}{\csc^2\theta}$ $\frac{\sec^2\theta}{\csc^2\theta} = \frac{\frac{1}{\cos^2\theta}}{\frac{1}{\sin^2\theta}} = \frac{\sin^2\theta}{\cos^2\theta} = \tan^2\theta$ 3) $\tan^2\theta$

4) $\cos\theta - \cos^3\theta$ 4) $\cos\theta\sin^2\theta$

$\cos\theta(1-\cos^2\theta) = \cos\theta\sin^2\theta$

Simplify the expression to either 1 or -1. Show work!

5) $\sin(x)\csc(-x)$ $\sin x \cdot \frac{1}{-\sin x} = -1$ 5) -1

6) $\cot(-x)\cot(\frac{\pi}{2} - x)$ 6) -1

$-\cot x(\tan x) = -1$

7) $\sin^2(-x) + \cos^2(-x)$ 7) 1

$\sin(-x)\cdot\sin(-x) + \cos(-x)(\cos(-x))$

$(-\sin x)(-\sin x) + (\cos x)(\cos x) = \sin^2 x + \cos^2 x = 1$

Simplify the expression to either a constant or a basic trig. function. Show work!

8) $\frac{\tan\left(\frac{\pi}{2} - x\right)\csc x}{\csc^2 x} = \frac{\cot x \cdot \csc x}{\csc^2 x} = \frac{\cot x}{\csc x} = \frac{\frac{\cos x}{\sin x}}{\frac{1}{\sin x}} = \cos x$ 8) $\cos x$

9) $(\sec^2 x + \csc^2 x) - (\tan^2 x + \cot^2 x)$ 9) 2

$\sec^2 x + \csc^2 x - \tan^2 x - \cot^2 x$

$(\sec^2 x - \tan^2 x) + (\csc^2 x - \cot^2 x)$

$1 + 1 = 2$

* Note: There can be multiple ways to simplify these expressions.

10) Use the basic identities to change the expression to one involving only sines and cosines. Then simplify to a basic trig. function. Show work!

10) $\sec x$

10) $(\sin x)(\tan x + \cot x) = \sin x \tan x + \sin x \cot x$

$= \sin x \cdot \frac{\sin x}{\cos x} + \sin x \cdot \frac{\cos x}{\sin x} = \frac{\sin^2 x}{\cos x} + \frac{\cos x}{1}\left(\frac{\cos x}{\cos x}\right) = \frac{\sin^2 x + \cos^2 x}{\cos x} = \frac{1}{\cos x} = \sec x$

11) $\sin x \cos x \tan x \sec x \csc x$

11) $\tan x$

$\frac{\sin x}{1} \cdot \frac{\cos x}{1} \cdot \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x} \cdot \frac{1}{\sin x} = \frac{\sin x}{\cos x} = \tan x$

12) $\frac{\tan x}{\csc^2 x} + \frac{\tan x}{\sec^2 x}$

12) $\tan x$

$\frac{\frac{\sin x}{\cos x}}{\frac{1}{\sin^2 x}} + \frac{\frac{\sin x}{\cos x}}{\frac{1}{\cos^2 x}} = \frac{\sin x}{\cos x} \cdot \frac{\sin^2 x}{1} + \frac{\sin x}{\cos x} \cdot \frac{\cos^2 x}{1} = \frac{\sin x(\sin^2 x + \cos^2 x)}{\cos x} = \tan x$

Combine the fractions and simplify to a multiple of a power of a basic trig. function(e.g. 3 $\tan^2 x$). Show work!

13) $\frac{1}{\sin^2 x} + \frac{\sec^2 x}{\tan^2 x} = \frac{1}{\sin^2 x} + \frac{1}{\cos^2 x} \cdot \frac{\cos^2 x}{\sin^2 x} = \frac{1}{\sin^2 x} + \frac{1}{\sin^2 x}$

$= \frac{2}{\sin^2 x} = 2\csc^2 x$

13) $2\csc^2 x$

14) $\frac{\sec x}{\sin x} - \frac{\sin x}{\cos x} = \frac{1}{\cos x} \cdot \frac{1}{\sin x} - \frac{\sin x}{\cos x} = \frac{1}{\cos x \sin x} - \frac{\sin x}{\cos x}\left(\frac{\sin x}{\sin x}\right)$

$= \frac{1 - \sin^2 x}{\cos x \sin x} = \frac{\cos^2 x}{\cos x \sin x} = \frac{\cos x}{\sin x} = \cot x$

14) $\cot x$

Write each expression in factored form as an algebraic expression of a single trigonometric function. Show work!

15) $\cos^2 x + 2\cos x + 1$

$(\cos x + 1)(\cos x + 1) = (\cos x + 1)^2$

15) $(\cos x + 1)^2$

16) $1 - 2\sin x + (1 - \cos^2 x)$

$1 - 2\sin x + \sin^2 x = \sin^2 x - 2\sin x + 1 = (\sin x - 1)^2$

16) $(\sin x - 1)^2$

17) $\cos x - 2\sin^2 x + 1$

$\cos x - 2(1 - \cos^2 x) + 1 = \cos x - 2 + 2\cos^2 x + 1$

$= 2\cos^2 x + \cos x - 1 = (2\cos x - 1)(\cos x + 1)$

17) $(2\cos x - 1)(\cos x + 1)$

18) $4\tan^2 x - \left(\frac{4}{\cot x}\right)(\sin x \csc x) + 1$

$4\tan^2 x - 4\tan x + 1 = (2\tan x - 1)^2$

18) $(2\tan x - 1)^2$

19) **Write each expression as an algebraic expression of a single trig. function (e.g. 2 sin x + 3). Show work!**

19) $\frac{1-\sin^2 x}{1+\sin x} = \frac{(1-\sin x)(1+\sin x)}{1+\sin x} = 1-\sin x$

19) $1-\sin x$

20) $\frac{\sin^2 x}{1+\cos x} = \frac{1-\cos^2 x}{1+\cos x} = \frac{(1+\cos x)(1-\cos x)}{1+\cos x} = 1-\cos x$

20) $1-\cos x$

Find all solutions in the interval [0, 2π). You do not need a calculator. Show work! (Factor where possible).

21) $2\cos x \sin x - \cos x = 0$

$\cos x(2\sin x - 1) = 0$

$\cos x = 0$ | $2\sin x - 1 = 0$

$x = \pi/2, 3\pi/2$ | $\sin x = 1/2$

$x = \pi/6, 5\pi/6$

21) $x = \pi/2, 3\pi/2, \pi/6, 5\pi/6$

22) $\tan x \sin^2 x = \tan x$

$\tan x \sin^2 x - \tan x = 0$

$\tan x(\sin^2 x - 1) = 0$

$\tan x = 0$ | $\sin^2 x - 1 = 0$

$\frac{\sin x}{\cos x} = 0$ | $\sin^2 x = 1$

$x = 0, \pi$ | $\sin x = \pm 1$

$x \neq \pi/2, 3\pi/2$ undefined here. | $x = \pi/2, 3\pi/2$

22) $x = 0, \pi$

23) $\tan^2 x = 3$

$\tan^2 x = 3$

$\tan x = \pm\sqrt{3}$ $x = \pi/3, 2\pi/3, 4\pi/3, 5\pi/3$

23) $x = \pi/3, 2\pi/3, 4\pi/3, 5\pi/3$

Find all solutions to the equation. You do not need a calculator. Show work! $[0, 2\pi)$

24) $4\cos^2 x - 4\cos x + 1 = 0$

$(2\cos x - 1)^2 = 0$ $2\cos x - 1 = 0$ $\cos x = 1/2$ $x = \pi/3, 5\pi/3$

24) $x = \pi/3, 5\pi/3$

25) $\cos(\sin x) = 1$

Think as $\cos\theta = 1$, where $\sin x = 0$.

$\theta = 0$, so, $\sin x = 0 \Rightarrow x = 0, \pi$

25) $x = 0, \pi$

26) $\sin^2 x - 2\sin x = 0$

$\sin x(\sin x - 2) = 0$ $\sin x = 0$ $\sin x = 2$

$x = 0, \pi$

26) $x = 0, \pi$

Find all solutions to the equation. $[0, 2\pi)$

27) $\cos x = 0.37$

(Use a calculator. Express your answer in radians, as a decimal rounded to the nearest thousandth.)

$x = \cos^{-1}(0.37) = 1.192$

$\text{or } 2\pi - 1.192 = 5.091$

27) $x = 1.192$ or 5.091

28) $\sin x = 0.30$

(Use a calculator. Express your answer in radians, as a decimal rounded to the nearest thousandth.)

$X = \sin^{-1}(.30) = .305$

& $\pi - .305 \quad 2.837$

28) $X = .305$ & 2.837

29) $\cos^2 x = 0.4$

(Use a calculator. Express your answer in radians as a decimal rounded to the nearest thousandth.)

$\cos x = \pm\sqrt{.4}$

$X = \cos^{-1}(\sqrt{.4})$ & $X = \cos^{-1}(-\sqrt{.4})$

$X = .886$ $\quad X = 2.256$

$+2\pi - .886 = 5.397$ $\quad$ & 4.028

29) _____

$X = .886, 2.256,$

4.028 & 5.397